# The weibull prior distribution on the information-based approach asset pricing model by brody hughston macrina 

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#### Abstract

This paper studied about the information-based approach asset pricing model. It is constructed by Brody,Hughston, and Macrina (BHM). Then it is called the BHM model or the BHM approach. In the BHM model, the asset price is defined as the value of cash flow at recent time based on the information in the market. The BHM model is constructed for the single cash flow case of an asset paying a single dividend at fixed time. Explicitly, the model is given by expectation of the upcoming dividend conditional at the market information filtration under the risk neutral probability measure. The information flow model containing the market information filtration is modelled as sum of information about dividends that they have been payed at past time (it is known the true information about dividends) and the noise information that there are in the market at recent time about the paying of the upcoming dividend. Using of Bayes formula is precisely to ascertain the value of cash flow ( the asset pricing ) at recent time based on the information about the dividends structure that have been payed at past time (as prior distribution) and the noise information in the market about paying of the upcoming dividend. Furthermore, the structure dividends that they have been payed at past time ( as prior distribution) will ascertain the solution of the BHM model. Brody, Hughston, and Macrina have found the closed form solution to prior distribution i.e. Exponential distribution and Gamma distribution. This paper will express to prior distribution i.e Weibull distribution.


Keywords : The BHM model, The Weibull prior distribution

## 1. Introduction

Once of purpose which people or investors invest will be to survive and increase their value of asset. There are more kind of types invesment which they can be used by people or investors to survive and increase their value of asset. Share is once of the more kind of types investment. Investors must to have the expectation to obtain dividends when they have invested their modal
in the share form. Dividend is proportion of profit that are divided for shareholders and they are proportional to the number of thread of their share. Therefore, dividend reflects the value of cash flow from an asset of share i.e. represents the company acceptances and it is also be represents the company expenseas when the company does to pay dividend to investors. While the investors position has invested their modal at the chosen of type share so investors can ascertain their value of cash flow or their value of asset at recent time based on the information about dividends have been payed in the past time and the noise information circulating in the market at recent time about the paying of the upcoming dividends.

To depart from phenomenon as above then Brody, Hughston, and Macrina constructed explicitly the information-based approach asset pricing model. Mathematically, the information-based approach asset pricing model by Brody,Hughston, and Macrina is defined as the asset price or the value of cash flow at recent time is the expectation of the upcoming dividend conditional the information filtration at recent time under the risk neutral probability measure. Because it represents the conditional expectation so using of Bayes formula is precisely to ascertain the asset price or the value of cash flow at recent time for the BHM model. Therefore the term of prior distribution or the dividends structure that have been payed at past time will also ascertain the result of asset price or the value of cash flow of the BHM model. For case of the certain dividends structure ( the certain prior distribution ) will result the closed form solution to the asset price or the value of cash flow of the BHM model. Brody, Hughston, and Macrina have artificially found the closed form solution to the Exponential prior distribution and the Gamma prior distribution. This paper expressed the closed form solution for the dividends stucture or the prior distribution i.e. Weibull distribution with the scale parameter $\delta$ and the shape parameter $\beta=2$. It is also called the Rayleigh distribution. The closed form solution is obtained by using of Gaussian Integration.

## 2. Constructing of the BHM model

The BHM model is a model to ascertain the asset price or the value of cash flow at recent time based on the market information. For case i.e. cash flows are the paying of dividend of a associated asset with equity, so the asset price or the value of cash flow is given by the upcoming dividend expectation conditional with the information filtration in the market. Therefore the asset pricing model or the value of cash flow model $\mathrm{S}_{\mathrm{t}}$ can be written explicitly in $[5,6]$ and $[8,12]$ as follows

$$
\begin{equation*}
\mathrm{S}_{\mathrm{t}}=\mathrm{P}_{\mathrm{tT}} \mathrm{E}^{\mathbb{Q}}\left[\mathrm{D}_{\mathrm{T}} \mid \mathcal{F}_{\mathrm{t}}\right] \tag{1}
\end{equation*}
$$

$\mathrm{S}_{\mathrm{t}}$ is the value of cash flow at time $\mathrm{t}, 0 \leq \mathrm{t}<\mathrm{T}$ from the asset paying a single dividend $D_{T}$ at fixed time $T$.

Modelling of information flow is constructed from available of information in the market about cash flow or dividend. The information is contained by a process $\left\{\xi_{t}\right\}_{0 \leq \leq \leq T}$ is defined in $[5,6]$ and $[7,12]$ as follows

$$
\begin{equation*}
\xi_{\mathrm{t}}=\sigma \mathrm{t} \mathrm{D}_{\mathrm{T}}+\beta_{\mathrm{tT}} \tag{2}
\end{equation*}
$$

Thus $\left\{\xi_{t}\right\}$ is called the market information process. This process compose from two parts i.e. $\sigma t \mathrm{D}_{\mathrm{T}}$ is the true information about dividends and process $\left\{\beta_{\mathrm{tT}}\right\}_{0 \leq \leq \leq T}$ is a standard Brownian Bridge over the time interval [0,T], so it takes zero values at time 0 and T. Thus process $\left\{\beta_{\mathrm{tT}}\right\}$ is the Gaussian process, $\beta_{0 \mathrm{~T}}=0, \beta_{\mathrm{TT}}=0, \beta_{\mathrm{tT}}$ is random variable have mean zero ( $\beta_{\mathrm{tT}}=0$ ) and $\operatorname{cov}\left(\beta_{\mathrm{sT}}, \beta_{\mathrm{tT}}\right)=\mathrm{s}(\mathrm{T}-\mathrm{t}) / \mathrm{T}$, for every $\mathrm{s} \leq \mathrm{t}$. ( see $\left.[7,8]\right)$

Assuming the market filtration $\left\{\mathcal{F}_{t}\right\}$ is resulted from the market information filtration $\mathcal{F}_{\mathrm{t}}=\sigma\left(\left\{\xi_{\mathrm{t}}\right\}_{0 \leq s \leq t}\right)$. Whereas the dividend $\mathrm{D}_{\mathrm{T}}$ is $\mathcal{F}_{\mathrm{T}}$-measurable but not $\mathcal{F}_{\mathrm{t}}$-measurable, for every $\mathrm{t}<\mathrm{T}$, so that the value of $\mathrm{D}_{\mathrm{T}}$ is known at time T but is not known at earlier time T.( see [7, 8]). For random variable $\mathrm{D}_{\mathrm{T}}=\mathrm{x}$ where it has the continue distribution, so as in [6]

$$
\begin{equation*}
\mathrm{E}^{\mathbb{Q}}\left[\mathrm{D}_{\mathrm{T}} \mid \xi_{\mathrm{t}}\right]=\int_{0}^{\infty} \mathrm{x} \pi_{\mathrm{t}}(\mathrm{x}) \mathrm{dx} \tag{3}
\end{equation*}
$$

where $\pi_{t}(x)$ is the probability density of random variable $D_{T}$, that is

$$
\begin{equation*}
\pi_{\mathrm{t}}(\mathrm{x})=\frac{\mathrm{d}}{\mathrm{dx}} \mathbb{Q}\left(\mathrm{D}_{\mathrm{T}} \leq \mathrm{x} \mid \xi_{\mathrm{t}}\right) \tag{4}
\end{equation*}
$$

By using of Bayes formula ( see $[2,9]$ ) so $\pi_{\mathrm{t}}(\mathrm{x})$ can be expressed as follows

$$
\begin{equation*}
\pi_{\mathrm{t}}(\mathrm{x})=\frac{\mathrm{p}(\mathrm{x}) \rho\left(\xi_{\mid} \mathrm{D}_{\mathrm{T}}=\mathrm{x}\right)}{\rho\left(\xi_{\mathrm{t}}\right)} \tag{5}
\end{equation*}
$$

where $\rho\left(\boldsymbol{\xi}_{\mathrm{t}}\right)$ and $\rho\left(\boldsymbol{\xi}_{\mathrm{t}} \mid \mathrm{D}_{\mathrm{t}}=\mathrm{x}\right)$ represent the probability density function and the conditional probability density function of random variable $\xi_{\mathrm{t}}$ and random variable $\xi_{\mathrm{t}}$ given $\mathrm{D}_{\mathrm{T}}=\mathrm{x}$, respectively.
Probability density function of random variable $\boldsymbol{\xi}_{\mathrm{t}}$ can be expressed as follows :

$$
\begin{equation*}
\rho\left(\xi_{\mathrm{t}}\right)=\int_{-\infty}^{\infty} \rho\left(\xi_{\mathrm{t}} \mid D_{\mathrm{T}}=\mathrm{x}\right) \mathrm{p}(\mathrm{x}) \mathrm{dx}=\int_{0}^{\infty} \rho\left(\xi_{\mathrm{t}} \mid D_{\mathrm{T}}=\mathrm{x}\right) \mathrm{p}(\mathrm{x}) \mathrm{dx} \tag{6}
\end{equation*}
$$

So that $\pi_{\mathrm{t}}(\mathrm{x})$ is held

$$
\begin{equation*}
\pi_{\mathrm{t}}(\mathrm{x})=\frac{\mathrm{p}(\mathrm{x}) \rho\left(\xi_{\mathrm{t}} \mid \mathrm{D}_{\mathrm{T}}=\mathrm{x}\right)}{\int_{0}^{\infty} \rho\left(\xi_{\mathrm{t}} \mid \mathrm{D}_{\mathrm{T}}=\mathrm{x}\right) \mathrm{p}(\mathrm{x}) \mathrm{dx}} \tag{7}
\end{equation*}
$$

If $\beta_{\mathrm{tT}}$ is a random variable for every t , where $0 \leq \mathrm{t} \leq \mathrm{T}$, the conditional probability density function of random variable $\xi_{\mathrm{t}}$ given $\mathrm{D}_{\mathrm{T}}=\mathrm{x}$ will be the probability density with mean $\sigma$ tx and variance $\mathrm{t}(\mathrm{T}-\mathrm{t}) / \mathrm{T}$ as follows

$$
\begin{align*}
& \rho\left(\xi_{\mathrm{t}} \mid \mathrm{D}_{T}=\mathrm{x}\right)=\frac{1}{\sqrt{2 \pi} \sqrt{\frac{\mathrm{tT}-t)}{\mathrm{T}}}} \exp \left(-\frac{1}{2} \frac{\left(\xi_{\mathrm{t}}-\sigma \mathrm{tx}\right)^{2}}{\frac{\mathrm{t}(\mathrm{~T}-\mathrm{t})}{\mathrm{T}}}\right)  \tag{8}\\
& \rho\left(\xi_{\mathrm{t}} \mid \mathrm{D}_{T}=\mathrm{x}\right)=\sqrt{\frac{\mathrm{T}}{2 \pi(\mathrm{t}-\mathrm{t})}} \exp \left(-\frac{\left(\xi_{\mathrm{t}}-\sigma \mathrm{tx}\right)^{2} \mathrm{~T}}{2 \mathrm{t}(\mathrm{~T}-\mathrm{t})}\right) \tag{9}
\end{align*}
$$

Substitusion $\rho\left(\xi_{\mathrm{t}} \mid \mathrm{D}_{T}=\mathrm{x}\right)$ in to Bayes formula is obtained:

$$
\begin{align*}
& \pi_{t}(x)=\frac{p(x) \sqrt{\frac{T}{2 \pi(T-t)}} \exp \left(-\frac{\left(\xi_{t}-\sigma t x\right)^{2} T}{2 t(T-t)}\right)}{\int_{0}^{\infty} p(x) \sqrt{\frac{T}{2 \pi t(T-t)}} \exp \left(-\frac{\left(\xi_{t}-\sigma t x\right)^{2} T}{2 t(T-t)}\right) d x} \\
& =\frac{p(x) \sqrt{\frac{T}{2 \pi(T-t)}} \exp \left(-\frac{\left(\xi_{t}-\sigma t x\right)^{2} T}{2 t(T-t)}\right)}{\sqrt{\frac{T}{2 \pi t(T-t)}} \int_{0}^{\infty} p(x) \exp \left(-\frac{\left(\frac{\xi t}{} \frac{\left.\xi^{-\sigma t x}\right)^{2} T}{2 t(T-t)}\right) d x}{}\right.} \\
& =\frac{p(x) \exp \left(-\frac{\left(\xi_{t}-\sigma x\right)^{2} T}{2(T-t)}\right)}{\int_{0}^{\infty} p(x) \exp \left(-\frac{\left(\xi_{t}-\sigma x\right)^{2} T}{2 t(T-t)}\right) d x} \\
& =\frac{p(x) \exp \left(-\frac{\xi_{t}^{2}}{2 t(T-t)}\right) \exp \left(\frac{2 \sigma x \xi_{t} \xi_{t} T-\sigma^{2} x^{2} t^{2} T}{2 t(T-t)}\right)}{\int_{0}^{\infty} p(x) \exp \left(-\frac{\xi_{t}^{2}}{2 t(T-t)}\right) \exp \left(\frac{2 \sigma x \xi_{t} \xi_{t} T-\sigma^{2} x^{2} t^{2} T}{2 t(T-t)}\right) d x} \\
& =\frac{p(x) \exp \left(-\frac{\xi_{t}^{2}}{2(T-t)}\right) \exp \left(\frac{2 \sigma x \xi_{t}\left(T-\sigma^{2} x^{2} t^{2} T\right.}{2 t(T-t)}\right)}{\exp \left(-\frac{\xi_{t}^{2}}{2(T-t)}\right) \int_{0}^{\infty} p(x) \exp \left(\frac{2 \sigma x \xi_{t} T-\sigma^{2} x^{2} t^{2} T}{2 t(T-t)}\right) d x} \\
& =\frac{p(x) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{t}-\frac{1}{2} \sigma^{2} x^{2} t\right)\right)}{\int_{0}^{\infty} p(x) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{t}-\frac{1}{2} \sigma^{2} x^{2} t\right)\right) d x} \tag{10}
\end{align*}
$$

so $\mathrm{E}^{\mathbb{Q}}\left[\mathrm{D}_{\mathrm{T}} \mid \xi_{\mathrm{t}}\right]$ becomes as follows:

$$
\begin{align*}
& E^{\mathbb{Q}}\left[D_{T} \mid \xi_{t}\right]=\int_{0}^{\infty} x \frac{p(x) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{t}-\frac{1}{2} \sigma x^{2} t\right)\right)}{\int_{0}^{\infty} p(x) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{t}-\frac{1}{2} \sigma x^{2} t\right)\right) d x} d x  \tag{11}\\
& E^{\mathbb{Q}}\left[D_{T} \mid \xi_{t}\right]=\frac{\int_{0}^{\infty} x p(x) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{\xi_{t}}-\frac{1}{2} \sigma^{2} x^{2} t\right)\right) d x}{\int_{0}^{\infty} p(x) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{t}-\frac{1}{2} \sigma^{2} x^{2} t\right)\right) d x} \tag{12}
\end{align*}
$$

Finally, it is obtained the formula of the information-based approach asset pricing by Brody-Hughston-Macrina as follows :

$$
\begin{equation*}
S_{t}=P_{t T} \frac{\int_{0}^{\infty} x p(x) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{t}-\frac{1}{2} \sigma^{2} x^{2} t\right)\right) d x}{\int_{0}^{\infty} p(x) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{t}-\frac{1}{2} \sigma^{2} x^{2} t\right)\right) d x} \tag{13}
\end{equation*}
$$

## 3. The Weibull prior distribution on the BHM model

The prior distribution $\mathrm{p}(\mathrm{x})$ on the information-based approach asset pricing model by Brody-Hughston-Macrina in (13) as above refer to the dividends structure $\mathrm{D}_{\mathrm{T}}$ paying at past time T. Brody-Hughston-Macrina have found the closed form solution to the devidends structure $\mathrm{D}_{\mathrm{T}}$ that has the Exponential distribution with parameter $\delta$. Then it is sometimes written $\operatorname{EXP}(\delta)$. Prior distribution $\mathrm{p}(\mathrm{x})$ for $\operatorname{EXP}(\delta)$ can be expressed in [12] as follows

$$
\begin{equation*}
\mathrm{p}(\mathrm{x})=\frac{1}{\delta} \exp \left(-\frac{\mathrm{x}}{\delta}\right) \tag{14}
\end{equation*}
$$

Thus the devidends structure $\mathrm{D}_{\mathrm{T}}$ has the Gamma distribution with parameter $\delta$ and positive integer n , it is sometimes written $\operatorname{GAM}(\delta, \mathrm{n})$ and the prior distribution $\mathrm{p}(\mathrm{x})$ as in [12] as follows

$$
\begin{equation*}
\mathrm{p}(\mathrm{x})=\frac{\delta^{\mathrm{n}}}{(\mathrm{n}-1)!} \mathrm{x}^{\mathrm{n}-1} \exp (-\delta \mathrm{x}) \tag{15}
\end{equation*}
$$

Hence, The BHM model also results the closed form solution.

The Closed form solution for the dividends structure $\mathrm{D}_{\mathrm{T}}$ or the prior distribution is the Exponential distribution with parameter $\delta$ in [12] as follows

$$
\begin{equation*}
\mathrm{S}_{\mathrm{t}}=1_{\{\mathrm{t}<\mathrm{T}\}} \mathrm{P}_{\mathrm{tT}}\left[\frac{\exp \left(-\frac{1}{2} \mathrm{~B}_{\mathrm{t}}^{2} / \mathrm{A}_{\mathrm{t}}\right)}{\sqrt{2 \pi \mathrm{AA}_{\mathrm{t}}} N\left(\mathrm{~B}_{\mathrm{t}} / \mathrm{A}_{\mathrm{t}}\right)}+\frac{\mathrm{B}_{\mathrm{t}}}{\mathrm{~A}_{\mathrm{t}}}\right] \tag{16}
\end{equation*}
$$

where $\mathrm{A}_{\mathrm{t}}=\sigma^{2} \frac{\mathrm{tT}}{(\mathrm{T}-\mathrm{t})}$, dan $\mathrm{B}_{\mathrm{t}}=\frac{\sigma \mathrm{T}}{(\mathrm{T}-\mathrm{t})} \xi_{\mathrm{t}}-\delta^{-1}$.
and the dividends structure $D_{T}$ or the prior distribution is the Gamma distribution with parameter $\delta$ and positive integer n so its closed form solution written in [12] as follows

$$
\begin{equation*}
S_{t}=1_{\{t<T\}} P_{t T} \frac{\sum_{k=0}^{n}\binom{n}{k} A_{t}^{\frac{1}{2} k-n} B_{t}^{n-k} F_{k}\left(-B_{t} / \sqrt{A_{t}}\right)}{\sum_{k=0}^{n-1}\binom{n-1}{k} A_{t}^{\frac{1}{2} k-n+1} B_{t}^{n-k-1} F_{k}\left(-B_{t} / \sqrt{A_{t}}\right)} \tag{17}
\end{equation*}
$$

where $A_{t}, B_{t}$ as above and generally, part of polinomial $\left\{F_{k}(x)\right\}_{k=0,1,2, \ldots}$ associated to Legendre polinomial.

This paper express the closed form solution to the dividends structure $\mathrm{D}_{\mathrm{T}}$ or the prior distribution has the Weibull distribution with the scale parameter $\delta$ and the shape parameter $\beta=2$. It is sometimes written $\operatorname{WEI}(\delta, 2)$. Thus, the Weibull prior distribution is of the form

$$
\begin{align*}
p(x) & =\frac{2}{\delta^{2}} x^{2-1} \exp \left(-\left(\frac{x}{\delta}\right)^{2}\right) \\
& =\frac{2}{\delta^{2}} x \exp \left(-\frac{x^{2}}{\delta^{2}}\right) \tag{18}
\end{align*}
$$

The closed form solution to the model in (1) for the Weibull prior distribution with the scale parameter $\delta$ and the shape parameter $\beta=2$ will be expressed in details as follows

$$
\begin{align*}
& S_{t}=P_{t T} \frac{\int_{0}^{\infty} x^{2} \exp \left(-\frac{x^{2}}{\delta^{2}}\right) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{t}-\frac{1}{2} \sigma^{2} x^{2} t\right)\right) d x}{\int_{0}^{\infty} x \exp \left(-\frac{x^{2}}{\delta^{2}}\right) \exp \left(\frac{T}{T-t}\left(\sigma x \xi_{t}-\frac{1}{2} \sigma^{2} x^{2} t\right)\right) d x}  \tag{19}\\
& S_{t}=P_{t T} \frac{\int_{0}^{\infty} x^{2} \exp \left(\frac{T}{T-t}\left(\sigma \xi_{t} x-\left(\frac{T-t}{T} \frac{1}{\delta^{2}}+\frac{1}{2} \sigma^{2}\right) x^{2}\right)\right) d x}{\int_{0}^{\infty} x \exp \left(\frac{T}{T-t}\left(\sigma \xi_{t} x-\left(\frac{T-t}{T} \frac{1}{\delta^{2}}+\frac{1}{2} \sigma^{2}\right) x^{2}\right)\right) d x} \tag{20}
\end{align*}
$$

$$
\begin{align*}
& S_{t}=P_{t T} \frac{\int_{0}^{\infty} x^{2} \exp \left(-\frac{T}{T-t}\left(\left(\frac{T-t}{T} \frac{1}{\delta^{2}}+\frac{1}{2} \sigma^{2} t\right) x^{2}-\sigma \xi_{\mathrm{t}} \mathrm{x}\right)\right) d \mathrm{dx}}{\int_{0}^{\infty} \mathrm{x} \exp \left(-\frac{T}{T-t}\left(\left(\frac{T-t}{T} \frac{1}{\delta^{2}}+\frac{1}{2} \sigma^{2} t\right) \mathrm{x}^{2}-\sigma \xi_{\mathrm{t}} \mathrm{x} x\right)\right) \mathrm{dx}}  \tag{21}\\
& \left.\left.S_{t}=P_{t T} \frac{\int_{0}^{\infty} x^{2} \exp \left(-\frac{T}{T-t}\left(x^{2}-\frac{\sigma \xi_{t}}{\left(\frac{T-t}{T} \frac{1}{\delta^{2}}+\frac{1}{2} \sigma^{2 t}\right)} x\right.\right.}{\int_{0}^{\infty} x \exp \left(-\frac{T}{T-t}\left(x^{2}-\frac{\sigma \xi_{t}}{\left(\frac{T-t}{T} \frac{1}{\delta^{2}}+\frac{1}{2} \sigma^{2} t\right.} x\right.\right.}\right)\right) d x  \tag{22}\\
& S_{t}=P_{t T} \frac{\int_{0}^{\infty} x^{2} \exp \left(-\frac{T}{T-t}\left(x^{2}-\frac{\sigma \xi_{t}}{\frac{1}{2}\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right.} x\right)\right) d x}{\int_{0}^{\infty} x \exp \left(-\frac{T}{T-t}\left(x^{2}-\frac{\sigma \xi_{t}}{\frac{1}{2}\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right.} x\right)\right) d x}  \tag{23}\\
& S_{t}=P_{t T} \frac{\left.\int_{0}^{\infty} x^{2} \exp \left(-\frac{T}{T-t}\left(x^{2}-\frac{2 \sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right.}\right)^{2} x+\left(\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right)}\right)^{2}-\left(\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right)}\right)^{2}\right)\right) d x}{\int_{0}^{\infty} x \exp \left(-\frac{T}{T-t}\left(x^{2}-\frac{2 \sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2}\right)^{2}} x+\left(\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right)}\right)^{2}-\left(\frac{\sigma \sigma \xi_{t}}{\left(\frac{2(T-t}{T \delta^{2}}+\sigma^{2} t\right)}\right)^{2}\right)\right) d x}  \tag{24}\\
& S_{t}=P_{t T} \frac{\int_{0}^{\infty} x^{2} \exp \left(-\frac{T}{T-t}\left(x-\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right)}\right)^{2}\right) \exp \left(\frac{T}{T-t}\left(\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}+\sigma^{2} t}\right)}\right)^{2}\right) d x}{\int_{0}^{\infty} x \exp \left(-\frac{T}{T-t}\left(x-\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right)}\right)^{2}\right) \exp \left(\frac{T}{T-t}\left(\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right.}\right)^{2}\right) d x}
\end{align*}
$$

$$
\begin{gather*}
S_{t}=P_{t T} \frac{\int_{0}^{\infty} x^{2} \exp \left(-\frac{T}{T-t}\left(x-\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right)}\right)^{2}\right) d x}{\left.\int_{0}^{\infty} x \exp \left(-\frac{T}{T-t}\left(x-\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right.}\right)^{2}\right)\right) d x}  \tag{26}\\
S_{t}=P_{t T} \frac{\int_{0}^{\infty} x^{2} \exp \left(-a(x-b)^{2}\right) d x}{\int_{0}^{\infty} x \exp \left(-a(x-b)^{2}\right) d x}, \text { where } a=\frac{T}{T-t}, b=\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right)}  \tag{27}\\
S_{t}=P_{t T} \frac{\int_{0}^{\infty}\left((x-b)^{2}+2 b x-b^{2}\right) \exp \left(-a(x-b)^{2}\right) d x}{\int_{0}^{\infty}((x-b)+b) \exp \left(-a(x-b)^{2}\right) d x}  \tag{28}\\
S_{t}=P_{t T} \frac{\int_{0}^{\infty}\left(y^{2}+2 b(y+b)-b^{2}\right) \exp \left(-a y^{2}\right) d y}{\int_{0}^{\infty}(y+b) \exp \left(-a y^{2}\right) d y}, w h e r e y=x-b \geq 0  \tag{29}\\
S_{t}=P_{t T} \frac{\int_{0}^{\infty} y^{2} \exp \left(-a y^{2}\right) d y+2 b \int_{0}^{\infty} y \exp \left(-a y^{2}\right) d y+b^{2} \int_{0}^{\infty} \exp \left(-a y^{2}\right) d y}{\int_{0}^{\infty} y \exp \left(-a y^{2}\right) d y+b \int_{0}^{\infty} \exp \left(-a y^{2}\right) d y}, a>0 \tag{30}
\end{gather*}
$$

To follow the integrals involving the exponential of a quadratic in [15] so is obtained

$$
\begin{equation*}
S_{t}=P_{t T} \frac{\frac{1}{2} \frac{(2.1-1)}{(2 a)^{1}} \sqrt{\frac{\pi}{a}}+2 b \int_{0}^{\infty} y \exp \left(-a y^{2}\right) d y+b^{2} \cdot \frac{1}{2} \sqrt{\frac{\pi}{a}}}{\int_{0}^{\infty} y \exp \left(-a y^{2}\right) d y+b \cdot \frac{1}{2} \sqrt{\frac{\pi}{a}}} \tag{31}
\end{equation*}
$$

Thus $\int_{0}^{\infty} y \exp \left(-a y^{2}\right) d y$ in equation (2) can be found by using of partial integrals (see [10] ) as follows :

$$
\begin{align*}
& \text { Let } u=\exp \left(-a y^{2}\right) \text {, so that } d u=-2 a y \exp \left(-a y^{2}\right) d y  \tag{32}\\
& \qquad d v=y d y \text {, so that } v=\int y d y=\frac{1}{2} y^{2} \tag{33}
\end{align*}
$$

so it is held
$\int_{0}^{\infty} \exp \left(-a y^{2}\right) y d y=\frac{1}{2} y^{2} \exp \left(-a y^{2}\right)-\int_{0}^{\infty} \frac{1}{2} y^{2}(-2 a y) \exp \left(-a y^{2}\right) d y$

$$
\begin{equation*}
=\frac{1}{2} y^{2} \exp \left(-a y^{2}\right)-a \int_{0}^{\infty} y^{3} \exp \left(-a y^{2}\right) d y \tag{34}
\end{equation*}
$$

and $\int_{0}^{\infty} y^{3} \exp \left(-\mathrm{a}^{2}\right) \mathrm{dy}$ can be obtained that is,

$$
\begin{equation*}
\text { Let } \mathrm{w}=\mathrm{y}^{2} \text {, so that } \mathrm{dw}=2 \mathrm{y} \mathrm{dy} \tag{35}
\end{equation*}
$$

So it is held

$$
\begin{align*}
\int_{0}^{\infty} y^{3} \exp \left(-a y^{2}\right) d y & =\frac{1}{2} \int_{0}^{\infty} y^{2} \exp \left(-a y^{2}\right) 2 y d y \\
& =\frac{1}{2} \int_{0}^{\infty} w \exp (-a w) d w  \tag{36}\\
\int_{0}^{\infty} y^{3} \exp \left(-a y^{2}\right) d y & =\frac{1}{2} \int_{0}^{\infty} \frac{1}{a^{2}}(a w) \exp (-a w) d(a w) \\
& =\frac{1}{2} \cdot \frac{1}{a^{2}} \int_{0}^{\infty}(a w) \exp (-a w) d(a w) \\
& =\frac{1}{2} \cdot \frac{1}{a^{2}} \cdot 1! \\
& =\frac{1}{2 a^{2}} \tag{37}
\end{align*}
$$

Thus the equation in (3) is held as follows

$$
\int_{0}^{\infty} y \exp \left(-a y^{2}\right) d y=\frac{1}{2} y^{2} \exp \left(-a y^{2}\right)-
$$

$a \int_{0}^{\infty} y^{3} \exp \left(-a y^{2}\right) d y$

$$
\begin{align*}
& =\frac{1}{2} y^{2} \exp \left(-a y^{2}\right)-a \cdot \frac{1}{2 a^{2}} \\
& =\frac{1}{2} y^{2} \exp \left(-a y^{2}\right)-\frac{1}{2 \mathrm{a}} \\
& =\frac{1}{2}\left(y^{2} \exp \left(-a y^{2}\right)-\frac{1}{a}\right) \tag{38}
\end{align*}
$$

Finally, the result of $S_{t}$ can be obtained as follows :

$$
\begin{equation*}
S_{t}=P_{t T} \frac{\frac{1}{2} \frac{(2.1-1)}{(2 a)^{1}} \sqrt{\frac{\pi}{a}}+2 b\left(\frac{1}{2}\left(y^{2} \exp \left(-a y^{2}\right)-\frac{1}{a}\right)\right)+b^{2} \cdot \frac{1}{2} \sqrt{\frac{\pi}{a}}}{\frac{1}{2}\left(y^{2} \exp \left(-a y^{2}\right)-\frac{1}{a}\right)+b \cdot \frac{1}{2} \sqrt{\frac{\pi}{a}}} \tag{39}
\end{equation*}
$$

$$
\begin{equation*}
S_{\mathrm{t}}=P_{\mathrm{tT}} \frac{\frac{1}{4 a} \sqrt{\frac{\pi}{a}}+\mathrm{by}^{2} \exp \left(-a y^{2}\right)-\frac{b}{a}+\frac{1}{2} b^{2} \sqrt{\frac{\pi}{a}}}{\frac{1}{2} y^{2} \exp \left(-a y^{2}\right)-\frac{1}{2 a}+\frac{1}{2} b \sqrt{\frac{\pi}{a}}} \tag{40}
\end{equation*}
$$

where $y=x-b, a=\frac{T}{T-t}, b=x-\frac{\sigma \xi_{t}}{\left(\frac{2(T-t)}{T \delta^{2}}+\sigma^{2} t\right)}$
The final result of the asset pricing model $S_{t}$ can be expressed as follows :

$$
\begin{align*}
& =P_{t T} \frac{\frac{T-t}{4 T} \sqrt{\frac{\pi(T-t)}{T}}+\frac{T \delta^{2} \sigma \xi_{t}}{\left(2(T-t)+\sigma^{2} T T \delta^{2}\right)}\left(x-\frac{T \delta^{2} \sigma \xi_{t}}{\left(2(T-t)+\sigma^{2} T T \delta^{2}\right)}\right)^{2} \exp \left(-\frac{T}{T-t}\left(x-\frac{T \delta^{2} \sigma \xi_{t}}{\left(2(T-t)+\sigma^{2} t T \delta^{2}\right)}\right)^{2}\right)-\frac{\left(\frac{T \delta^{2} \sigma \xi_{t}}{\left(2(T-t)+\sigma^{2} T T \delta^{2}\right)}\right)(T-t)}{T}+\frac{1}{2}\left(\frac{T \delta^{2} \sigma \xi_{t}}{\left(2(T-t)+\sigma^{2} T \delta^{2}\right)}\right)^{2} \sqrt{\frac{\pi(T-t)}{T}}}{\frac{1}{2}\left(x-\frac{T \delta^{2} \sigma \xi_{t}}{\left(2(T-t)+\sigma^{2} T T \delta^{2}\right)}\right)^{2} \exp \left(-\frac{T}{T-t}\left(x-\frac{T \delta^{2} \sigma \xi_{t}}{\left(2(T-t)+\sigma^{2} t T \delta^{2}\right)}\right)^{2}\right)-\frac{(T-t)}{2 T}+\frac{1}{2} \frac{T \delta^{2} \sigma \xi_{t}}{\left(2(T-t)+\sigma^{2} t T \delta^{2}\right)} \sqrt{\frac{\pi(T-t)}{T}}} \tag{41}
\end{align*}
$$

## 4. Conclusion

The information-based approach asset pricing model by Brody-HughstonMacrina modelled the asset price and the value of cash flow represents the expectation of the upcoming dividend conditional at the market information under the risk neutral probability measure. Thus the model is called the BHM model or the BHM approach. Using of Bayes formula is very precise to find the solution of this model. Somewhat artificial, Brody-Hughston-Macrina have ascertained for an Exponentiall distributed payout or the dividends structure has the Exponential distribution or the other words, its prior distribution are the Exponential distribution results the closed form solution. Brody-HughstonMacrina also find the closed from solution to the Gamma prior distribution. This paper express the closed form solution to the Weibull prior distribution.

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