A Black-Scholes Model from an Information-Based Perspective by Brody Hughston Macrina

Mutijah
1. Ph.D Student of Mathematics
Gadjah Mada University
Yogyakarta, Indonesia
2. STAIN Purwokerto
Banyumas, Central Java, Indonesia
mutipurwokerto@yahoo.com

Suryo Guritno
Department of Mathematics
Gadjah Mada University
Yogyakarta, Indonesia
suryoguritno@ugm.ac.id

Gunardi
Department of Mathematics
Gadjah Mada University
Yogyakarta, Indonesia
gunardi@ugm.ac.id

***Abstract* :** *The information-based approach asset pricing model by Brody-Hughston-Macrina is constructed based on the dividends information that can be accessed by investors at recent time in a financial market. Brody-Hughston-Macrina modeled the asset pricing at recent time for case of the cash flows value paying a single dividend at the certain time. The model is given by the expectation of the upcoming dividends conditional at the market information filtration under the risk neutral probability measure. The market information filtration is contained by the recent information flows that represent sum of the two of information i.e. the true information of dividends and the noise information in the financial market. Brody-Hughston-Macrina also constructed a model from setting that is a limited-liability asset that pays no interim dividends and is sold off at time fixed for the certain value. In this model, the recent information flows represent the sum of the market factor information and the noise information. Then this model is called a Black-Scholes Model from an Information-Based Perspective. Brody-Hughston-Macrina did imprecision in using of Gaussian Integrals to obtain the final result of the Black-Scholes Model from an Information-Based Perspective. This paper explains an imprecision for using of Gaussian Integrals in deriving the final result of the Black-Sholes Model from an Informative-Perspective, so it is found a different final model from the final model by Brody-Hughston-Macrina.*

Keywords: *The Information-Based Approach Asset Pricing, Dividend, the Information Flows Model, Gaussian Integrals, The Black-Scholes Model from an Information-Based Perspective.*

I. INTRODUCTION

An approach of the asset pricing model is built by Brody, Hughston, and Macrina. They constructed the asset pricing model based on the type of the accessible information in financial market. Further, this model is called an information-based asset pricing model by Brody-Hughston-Macrina or this model is usually called the BHM Model or the BHM approach. According to the BHM approach, asset price dynamics are modeled under the assumption that market participants do not have access to the information about the actual value of the relevant market variables. In the other words, it is known that the market participants acknowledge is only partial noisy information about the associated market factors. There are the two asset pricing models that are built by Brody-Hughston-Macrina. The two asset pricing models are built under the associated information of market factor, circulating in the financial market.

For the first model, Brody-Hughston-Macrina modeled the recent asset pricing is given by the expectation of the upcoming dividends conditional at the market information filtration under the risk neutral probability measure. The market information filtration is contained by the recent information flows. Brody-Hughston-Macrina modeled the recent information flows as sum of the two terms that is the true information about paying of dividends at years ago and a standard Brownian Bridge process that represents the pure noise information. This Brownian Bridge process is not adapted to the market filtration and it can not be accessed to market participants which reflects the fact that market participants can not perceive the true information without the noise in the financial market until the dividend is paid.

For the second model, Brody-Hughston-Macrina make the Black Scholes model set up from an information-based perspective that derived from the first model as above. The model set up is a limited-liability asset that pays no interim dividends and it is sold off at time fixed for the certain value. In this model, the recent information flows is modeled as sum of the true information about the associated market factors and the Brownian Bridge process that represents the pure noise information. From the first model by Brody-Hughston-Macrina and using of Gaussian Integrals then is obtained a Black Scholes Model from an Information-Based Perspective.

We find an imprecision for using of Gaussian Integrals in deriving of the final model of The Black-Scholes Model from Information-Based Perspective, so we find the different final model from the final model by Brody-Hughston-Macrina. This paper makes an effort to refer an imprecision of using of Gaussian Integrals successively.

II. AN INFORMATION-BASED ASSET PRICING MODEL BY BRODY-HUGHSTON-MACRINA

An Information-Based Asset Pricing Model represents once of model that is first proposed by Brody-Hughston-Macrina in Brody, et. al [3]. Further, it is called the BHM model or the BHM approach. This Model is formulated as follows; the asset price is given by the expectation of the upcoming dividends conditional at the market information filtration under the risk neutral probability measure. The market information filtration is contained by the recent information flows. For case, cash flows are the payout of the associated dividends of equity then is obtained the explisit model for the asset pricing S_t :

$$S_t = P_{tT}E^{\mathbb{Q}}[D_T|\mathcal{F}_t] \quad (1)$$

that means S_t is the value of cash flows at time t, $0 \leq t < T$ from asset that payout single dividend D_T at time T . In equation (1) as above, P_{tT} represents the discount factors that it is to be equal to $e^{-r(T-t)}$ with r is the interest rates. Then $\mathbb{Q}$ is the risk neutral probability, and $\mathcal{F}_t$ is the market information filtration.

Modelling the infornation flows based on assuming that the information available in market about dividends is contained by the process $\{\xi_t\}_{0\leq t\leq T}$ defining with :

$$\xi_t = \sigma t D_T + \beta_{tT} \quad (2)$$

$\{\xi_t\}$ is called the market information process. This process is compossed from the two parts that is tD_T standing as the true information about dividends, it grows in magnitude as t increases and the proses $\{\beta_{tT}\}_{0\leq t\leq T}$ is a standard Brownian Bridge on interval [0, T]. Thus $\{\beta_{tT}\}$ is Gaussian, $\beta_{0T} = 0$, $\beta_{TT} = 0$, mean of random variable $\beta_{tT} = 0$ dan $\text{cov}(\beta_{sT},\beta_{tT}) = s(T-t)/T$, for $s \leq t$.

The market filtration $\{\mathcal{F}_t\}$ is assumed to be equal to the filtration generated by the market information process $\{\xi_t\}$ that is $\mathcal{F}_t = \sigma(\{\xi_t\}_{0\leq s\leq t})$. Hence, dividend D_T is $\mathcal{F}_T$ – measurable but not $\mathcal{F}_t$-measurable for $t < T$, so that the value D_T is known at time T but is not known earlier. The incomplete information model as above is developed by Brody et al. [3] at first.

If random variable D_T to be equal to x having continuous distribution then,

$$E^{\mathbb{Q}}[D_T|\xi_t] = \int_0^\infty x\pi_t(x)\,dx \quad (3)$$

where $\pi_t(x)$ represents the conditional probability density function for the random variable D_T :

$$\pi_t(x) = \frac{d}{dx}\,\mathbb{Q}(D_T \leq x|\xi_t) \quad (4)$$

By using of the Bayes formula, $\pi_t(x)$ is expressed in Brody et al. [3], Brody et al. [4], Brody et al. [5], Caliskan,N [6], and Macrina,A [11] as follows

$$\pi_t(x) = \frac{p(x)\rho(\xi_t|D_T=x)}{\rho(\xi_t)} \quad (5)$$

where p(x) represents the priori probability density function for D_T which assumed as an initial condition. $(\boldsymbol{\xi}_t)$ and $\rho(\boldsymbol{\xi}_t|D_T = x)$ denotes the probability density function and the probability density function for the random variable $\boldsymbol{\xi}_t$ conditional $D_T = x$.

The probability density function for the random variable $\boldsymbol{\xi}_t$ can be expressed as follows

$$\rho(\xi_t) = \int_{-\infty}^{\infty} \rho(\xi_t|D_T = x)p(x)dx$$

$$= \int_0^{\infty} \rho(\xi_t|D_T = x)p(x)dx \quad (6)$$

So $\pi_t(x)$ in equation (5) is held

$$\pi_t(x) = \frac{p(x)\rho(\xi_t|D_T=x)}{\int_0^\infty \rho(\xi_t|D_T=x)p(x)dx} \quad (7)$$

If β_{tT} is a random variable for every $0\leq t\leq T$, the conditional probability density function for a random variable $\boldsymbol{\xi}_t$ with $D_T = x$ is the probability density with mean σtx dan variance $t(T-t)/T$

$$\rho(\xi_t|D_T = x) = \frac{1}{\sqrt{2\pi}\sqrt{\frac{t(T-t)}{T}}}\exp\left(-\frac{1}{2}\frac{(\xi_t - \sigma tx)^2}{\frac{t(T-t)}{T}}\right)$$

$$\rho(\xi_t|D_T = x) = \sqrt{\frac{T}{2\pi t(T-t)}}\ \exp\left(-\frac{(\xi_t - \sigma t x)^2\, T}{2t(T-t)}\right) \quad (8)$$

Substitution $\rho(\xi_t|D_T = x)$ into the Bayes formula is obtained :

$$\pi_t(x) = \frac{p(x)\sqrt{\frac{T}{2\pi t(T-t)}}\ \exp\left(-\frac{(\xi_t-\sigma t x)^2 T}{2t(T-t)}\right)}{\int_0^\infty p(x)\sqrt{\frac{T}{2\pi t(T-t)}}\ \exp\left(-\frac{(\xi_t-\sigma t x)^2 T}{2t(T-t)}\right) dx}$$

$$= \frac{p(x)\sqrt{\frac{T}{2\pi t(T-t)}}\ \exp\left(-\frac{(\xi_t-\sigma t x)^2 T}{2t(T-t)}\right)}{\sqrt{\frac{T}{2\pi t(T-t)}}\int_0^\infty p(x)\ \exp\left(-\frac{(\xi_t-\sigma t x)^2 T}{2t(T-t)}\right) dx}$$

$$= \frac{p(x)\exp\left(-\frac{(\xi_t-\sigma t x)^2 T}{2t(T-t)}\right)}{\int_0^\infty p(x)\exp\left(-\frac{(\xi_t-\sigma t x)^2 T}{2t(T-t)}\right) dx}$$

$$= \frac{p(x)\exp\left(-\frac{\xi_t^2}{2t(T-t)}\right)\exp\left(\frac{2\sigma x\xi_t tT - \sigma^2x^2t^2T}{2t(T-t)}\right)}{\int_0^\infty p(x)\exp\left(-\frac{\xi_t^2}{2t(T-t)}\right)\exp\left(\frac{2\sigma x\xi_t tT - \sigma^2x^2t^2T}{2t(T-t)}\right) dx}$$

$$= \frac{p(x)\exp\left(-\frac{\xi_t^2}{2t(T-t)}\right)\exp\left(\frac{2\sigma x\xi_t tT - \sigma^2x^2t^2T}{2t(T-t)}\right)}{\exp\left(-\frac{\xi_t^2}{2t(T-t)}\right)\int_0^\infty p(x)\exp\left(\frac{2\sigma x\xi_t tT - \sigma^2x^2t^2T}{2t(T-t)}\right) dx}$$

$$= \frac{p(x)\exp\left(\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right)}{\int_0^\infty p(x)\exp\left(\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right) dx} \quad (9)$$

So $E^{\mathbb{Q}}[D_T|\xi_t]$ is obtained as follows

$$E^{\mathbb{Q}}[D_T|\xi_t] = \int_0^\infty x\frac{p(x)\exp\left(\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right)}{\int_0^\infty p(x)\exp\left(\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right) dx}\ dx$$

$$= \frac{\int_0^\infty x\,p(x)\exp\left(\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right) dx}{\int_0^\infty p(x)\exp\left(\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right) dx} \quad (10)$$

Finally, the final result is obtained the formula of the information-based asset pricing model by Brody-Hughston-Macrina,that is

$$S_t = P_{tT}\frac{\int_0^\infty x\,p(x)\exp\left(\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right) dx}{\int_0^\infty p(x)\exp\left(\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right) dx} \quad (11)$$

III. A BLACK SCHOLES MODEL FROM AN INFORMATION-BASED PERSPECTIVE

Brody-Hughston-Macrina also make the model setting as follows; a limited-liability asset that pays no interim dividends and that at time T is sold off for the value S_T. S_T is log-normally distributed and is of the form

$$S_T = S_0\exp(rT - \tfrac{1}{2}\nu^2 T + \nu\sqrt{T}X_T) \quad (12)$$

where S_0, r, $\boldsymbol{\nu}$ are given constants and X_T is a standard normally distributed random variable. The corresponding information process is given by

$$\xi_t = \sigma t X_T + \beta_{tT} \quad (13)$$

The price proses $\{S_t\}_{0\le t\le T}$ is obtained from :

$$S_t = P_{tT}\,E^{\mathbb{Q}}(\Delta_T(X_T)|\mathcal{F}_t) \quad (14)$$

and the market information filtration $\mathcal{F}_t$ is contained by the market information flows

$$\xi_t = \sigma t X_T + \beta_{tT} \quad (15)$$

So for $t < T$, the equation S_t results :

$$S_t = P_{tT}\int_{-\infty}^{\infty}\Delta_T(x)\,\pi_{tT}(x)\,dx \quad (16)$$

Where by the Bayes formula is obtained $\pi_{tT}(x)$ as follows

$$\pi_{tT}(x) = \frac{p(x)\exp\left[\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right]}{\int_{-\infty}^{\infty}p(x)\exp\left[\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right] dx} \quad (17)$$

For this problem S_T plays the role of the single cash flow $\Delta_T(x)$ for $X_T = x$.
So it is obtained the equation S_t as follows

$$S_t = P_{tT}\int_{-\infty}^{\infty}S_0\exp\left(rT - \tfrac{1}{2}\nu^2T + \nu\sqrt{T}\,x\right)\frac{p(x)\exp\left[\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right]}{\int_{-\infty}^{\infty}p(x)\exp\left[\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right] dx}\ dx$$

$$S_t = P_{tT}\frac{\int_{-\infty}^{\infty}S_0\exp\left(rT - \frac{1}{2}\nu^2T + \nu\sqrt{T}\,x\right)p(x)\exp\left[\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right] dx}{\int_{-\infty}^{\infty}p(x)\exp\left[\frac{T}{T-t}\left(\sigma x\xi_t - \frac{1}{2}\sigma^2x^2t\right)\right] dx}$$

$$S_t = P_{tT}S_0\exp\left(rT - \tfrac{1}{2}\nu^2T\right)\frac{\int_{-\infty}^{\infty}p(x)\exp\left(\nu\sqrt{T}\,x\right)\exp\left[\frac{T}{T-t}\sigma\xi_t x - \frac{1}{2}\frac{T}{T-t}\sigma^2tx^2\right] dx}{\int_{-\infty}^{\infty}p(x)\exp\left[\frac{T}{T-t}\sigma\xi_t x - \frac{1}{2}\frac{T}{T-t}\sigma^2tx^2\right] dx}$$

$$S_t = P_{tT}S_0\exp\left(rT - \tfrac{1}{2}\nu^2T\right)\frac{\int_{-\infty}^{\infty}p(x)\exp\left[\frac{T}{T-t}\sigma\xi_t x + \nu\sqrt{T}\,x - \frac{1}{2}\frac{T}{T-t}\sigma^2tx^2\right] dx}{\int_{-\infty}^{\infty}p(x)\exp\left[\frac{T}{T-t}\sigma\xi_t x - \frac{1}{2}\frac{T}{T-t}\sigma^2tx^2\right] dx}$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \frac{\int_{-\infty}^{\infty} p(x) \exp\left[\left(\frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}\right)x - \frac{1}{2}\frac{T}{T-t}\sigma^2 t x^2\right] dx}{\int_{-\infty}^{\infty} p(x) \exp\left[\frac{T}{T-t}\sigma\xi_t x - \frac{1}{2}\frac{T}{T-t}\sigma^2 t x^2\right] dx} \qquad (18)$$

Because X_T is assumed the standard normally distributed then

$$p(x) = \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2} x^2) \qquad (19)$$

to follow the Gaussian Integrals in Straub [17] and Macrina, A [11], that is

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\tfrac{1}{2}x^2\right) \exp\left(ax - bx^2\right) dx = \frac{1}{\sqrt{2b+1}} \exp\left(\frac{1}{2}\frac{a^2}{2b+1}\right) \qquad (20)$$

then S_t is obtained as follows :

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \frac{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2) \exp\left[\left(\frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}\right)x - \frac{1}{2}\frac{T}{T-t}\sigma^2 t x^2\right] dx}{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2) \exp\left[\frac{T}{T-t}\sigma\xi_t x - \frac{1}{2}\frac{T}{T-t}\sigma^2 t x^2\right] dx}$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \frac{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \exp(-\frac{1}{2}x^2) \exp\left[\left(\frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}\right)x - \frac{1}{2}\frac{T}{T-t}\sigma^2 t x^2\right] dx}{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \exp(-\frac{1}{2}x^2) \exp\left[\frac{T}{T-t}\sigma\xi_t x - \frac{1}{2}\frac{T}{T-t}\sigma^2 t x^2\right] dx} \qquad (21)$$

The equation (21) is done partly. By using of Gaussian integrals is obtained the result of the equation (21) in its parts that is the integrals in the numerator in the expression of the equation (21) as above can be worked out explicitly as follows

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \exp(-\tfrac{1}{2}x^2) \exp\left[\left(\frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}\right)x - \frac{1}{2}\frac{T}{T-t}\sigma^2 t x^2\right] dx$$

$$= \frac{1}{\sqrt{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}} \exp\left(\frac{1}{2}\frac{\left(\frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}\right)^2}{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}\right) \qquad (22)$$

where $a = \frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}$, $b = \frac{1}{2}\frac{T}{T-t}\sigma^2 t$

and the integrals in denominator in the expression of the equation (21) as above can be also worked out explicitly as follows

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \exp(-\tfrac{1}{2}x^2) \exp\left[\frac{T}{T-t}\sigma\xi_t x - \frac{1}{2}\frac{T}{T-t}\sigma^2 t x^2\right] dx$$

$$= \frac{1}{\sqrt{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}} \exp\left(\frac{1}{2}\frac{\left(\frac{T}{T-t}\sigma\xi_t\right)^2}{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}\right) \qquad (23)$$

where $a = \frac{T}{T-t}\sigma\xi_t$, $b = \frac{1}{2}\frac{T}{T-t}\sigma^2 t$

Further, it is referred step by step in details to obtain the final result in the equation (21). The steps are referred clearly as follows

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \frac{\frac{1}{\sqrt{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}} \exp\left(\frac{1}{2}\frac{\left(\frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}\right)^2}{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}\right)}{\frac{1}{\sqrt{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}} \exp\left(\frac{1}{2}\frac{\left(\frac{T}{T-t}\sigma\xi_t\right)^2}{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}\right)}$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \frac{\exp\left(\frac{1}{2}\frac{\left(\frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}\right)^2}{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}\right)}{\exp\left(\frac{1}{2}\frac{\left(\frac{T}{T-t}\sigma\xi_t\right)^2}{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}\right)}$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \exp\left(\frac{1}{2}\frac{\left(\frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}\right)^2}{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1} - \frac{1}{2}\frac{\left(\frac{T}{T-t}\sigma\xi_t\right)^2}{2\left(\frac{1}{2}\frac{T}{T-t}\sigma^2 t\right)+1}\right)$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \exp\left(\frac{1}{2}\frac{1}{\frac{tT}{T-t}\sigma^2+1}\left(\left(\frac{T}{T-t}\sigma\xi_t + \nu\sqrt{T}\right)^2 - \left(\frac{T}{T-t}\sigma\xi_t\right)^2\right)\right)$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \exp\left(\frac{1}{2}\frac{1}{\frac{tT}{T-t}\sigma^2+1}\left(\left(\frac{T}{T-t}\sigma\xi_t\right)^2 + 2\frac{T}{T-t}\sigma\nu\sqrt{T}\,\xi_t + \left(\nu\sqrt{T}\right)^2 - \left(\frac{T}{T-t}\sigma\xi_t\right)^2\right)\right)$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \exp\left(\frac{1}{2}\frac{1}{\frac{tT}{T-t}\sigma^2+1}\left(2\frac{T}{T-t}\sigma\nu\sqrt{T}\,\xi_t + \nu^2 T\right)\right)$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \exp\left(\frac{1}{2}\frac{1}{\sigma^2\tau+1}\left(2\frac{T}{T-t}\sigma\nu\sqrt{T}\,\xi_t + \nu^2 T\right)\right),$$

where $\tau = \frac{tT}{(T-t)}$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \tfrac{1}{2}\nu^2 T\right) \exp\left(\frac{\frac{T}{T-t}\sigma\nu\sqrt{T}}{\sigma^2\tau+1}\xi_t + \frac{1}{2}\frac{1}{\sigma^2\tau+1}\nu^2 T\right)$$

$$S_t = P_{tT}\, S_0 \exp(rT)\exp\left(\frac{\frac{T}{T-t}\sigma\nu\sqrt{T}}{\sigma^2\tau+1}\xi_t - \frac{1}{2}\nu^2 T + \frac{1}{2}\frac{1}{\sigma^2\tau+1}\nu^2 T\right)$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \frac{1}{2}\left(1 - \frac{1}{\sigma^2\tau+1}\right)\nu^2 T + \frac{\frac{T}{T-t}\sigma\nu\sqrt{T}}{\sigma^2\tau+1}\xi_t\right)$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \frac{1}{2}\left(\frac{\sigma^2\tau+1}{\sigma^2\tau+1} - \frac{1}{\sigma^2\tau+1}\right)\nu^2 T + \frac{\frac{T}{T-t}\sigma\nu\sqrt{T}}{\sigma^2\tau+1}\xi_t\right)$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \frac{1}{2}\frac{\sigma^2\tau}{\sigma^2\tau+1}\nu^2 T + \frac{\frac{T}{T-t}\sigma\nu\sqrt{T}}{\sigma^2\tau+1}\xi_t\right)$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \frac{1}{2}\frac{\sigma^2\tau}{\sigma^2\tau+1}\nu^2 T + \frac{t\frac{T}{T-t}\sigma\nu\sqrt{T}}{t\,(\sigma^2\tau+1)}\xi_t\right)$$

$$S_t = P_{tT}\, S_0 \exp\left(rT - \frac{1}{2}\frac{\sigma^2\tau}{\sigma^2\tau+1}\nu^2 T + \frac{\sigma\tau\nu\sqrt{T}}{t(\sigma^2\tau+1)}\xi_t\right) \qquad (24)$$

We obtain the final result for a Black Scholes model from an information based perspective in equation (24) from the model set up by Brody-Hughston-Macrina, and the equation (24) refer the different final model from the final model by Brody-Hughston-Macrina in Macrina,A [11] in equation (6.16), page 80 as follows

$$S_t = P_{tT}\, S_0 \exp\left(rT - \frac{1}{2}\nu^2 T - \frac{1}{2}\frac{\nu\sqrt{T}}{\sigma^2\tau + 1} + \frac{\sigma\tau\, \nu\sqrt{T}}{t(\sigma^2\tau + 1)}\, \xi_t\right) \quad (25)$$

An imprecision is found in using of Gaussian Integrals to get the final result of a Black-Scholes Model from Information-based Perspective by Brody-Hughston-Macrina. We find an imprecision in the deriving step of Gaussian Integrals. Our result of analysis, an imprecision of using of Gaussian Integrals is in the fifth row in arrangements of our deriving steps of S_t in details as above. Precisely, Brody-Hughston-Macrina do an imprecision in the analysis of algebra trick of completing the square, that is referred in the fifth row in arrengements of our deriving steps of S_t in details as above .

This paper also explains to refer that the term structure both the final model in the equation (24) and in the equation (25). Its result is that the equation (24) has the term structure to be equal to the term structure of a Black Scholes model under the risk neutral probabilities in Ross, S.M.[9] and Higham,D.J.[8].

IV. CONCLUSION

This paper presented the two information-based asset pricing models by Brody-Hughston-Macrina. The two models are built based on the recent information flows circulating in the financial market. The first model involve the upcoming dividends and the recent information flows formed from sum of the two terms that is the true information about the upcoming dividends and the noise information represented the Brownian Bridge process.

The second model, Brody-Hughston-Macrina constructed a Black Scholes model from an information-based perspective with a certain model set up. The recent information flows model involve the market factor. The recent information flows are also formed from sum of the two terms, that is the true information about the market factors and the noise information represented the Brownian Bridge process.

By the similar model set up for the second model, we find the different model from the derived model by Brody-Hughston-Macrina. This difference is referred by an imprecision of using of Gaussian Integrals. Further research, we will do and extend the model in the equation (24) for option and the binomial model.

ACKNOWLEDGMENT

The first author is very grateful to Suryo Guritno and Gunardi, my supervisor, for their supports, encouragement, and for introducing me to the great knowledge of mathematical finance. The first author would also very grateful to Subanar, Sri Haryatmi, Abdurakhman, and Dedi Rosadi for their patiences to understand me in my material topics. The first author would also like to express my second gratitude to Suryo Guritno and Gunardi, my supervisor, and my second gratitude to Sri Haryatmi and Dedi Rosadi. They have also given correction for this paper in Monitoring and Evaluating for Doctor Program of Mathematics Gadjah Mada University in April 26-27, 2012. Finally, the author thank for Department of Mathematics Gadjah Mada University for the facilities that they are given us to our works.

REFERENCES

[1]Bain,L.J, and Max E,"Introduction to Probability and Mathematical Statistics",University of Missouri-Rolla:Duxbury Press,1992

[2]Box-Tiao,"Bayesian Inference in Statistical Analysis",Department of Statistics University of Wisconsin:Addison-Wesley Publising Company,1973

[3]Brody, D.C,Hughston,L.P, and Macrina,A,"Beyond Hazard Rates:a New Framework for Credit Risk Modelling", Working Paper, King's College London and Imperial College London (Version: July 7, 2005)

[4]Brody, D.C,Hughston,L.P, and Macrina,A."Information-Based Asset Pricing", Working Paper, King's College London and Imperial College London (Version: March 26, 2006)

[5]Brody, D.C,Hughston,L.P, and Macrina,A,"Modelling Information Flows in Financial Markets". Paper,King's College London, Imperial College London and Kyoto University Japan,2010

[6]Caliskan,N,"Asset Pricing Models:Stochastic Volatility and Information-Based Approaches",Thesis,Department of Financial Mathematics,February 2007

[7]Cochrane, J.H,"Asset Pricing",Chicago:Graduate School of Business University of Chicago,2000

[8]Higham,D.J,"An Introduction to Financial Option Valuation Mathematics,Stochastics and Computation,Department of Mathematics University of Strathclyde:Cambridge University Press,2004

[9]Karatzas, I and Shreve, S.E,"Brownian Motion and Stochastic Calculus Second Edition",New York:Springer-Verlag, Inc,1991

[10]Kellison,S.G,"The Theory of Interest Second Edition",Georgia State University:Richard D, Irwin,Inc,1991

[11]Macrina,A,"An Information-Based Framework for Asset Pricing:X-Factor Theory and its Applications", PhD Thesis, King's College London,2006

[12]Rolski, T, Schmidli,H, Schmidt, V, and Teugels,J,"Stochastic Processes for Insurance and Finance,New York:John Wiley & Sons, Inc,2000

[13]Ross,S.M,"Introduction to Probability Models Sixth Edition",Department of Industrial Engineering and

Operation Research University of California,Academic Press,1997

[14]Ross, S.M,"An Elementary Introduction to Mathematical Finance Options and Other Topics Second Edition",United States of America:Cambridge University Press,2003

[15]Ruppert,D,"Statistics and Finance an Introduction",United State of America:Springer-Verlag New York,LLC,2004

[16]Shreve,S.E,"Stochastic Calculus for Finance II Continuous-Time Models",United States of America:Springer Science+Business Media,Inc,2004

[17]Straub, W.O,"A Brief Look at Gaussian Integrals",Article:Pasadena California,2009